99 學年國立台灣大學機械系學士班申請入學考試試題

※ 考試時間：１００分鐘
不得參考圖書資料等

1. 請由下列方程式 $z^4+z^3+z^2+z+1=0$，解出複數 z。(20%)

2. 點 P(0,-5)並不在拋物線 $f(x)=x^2+2x-1$ 上，但是過點 P 可以找出與拋物線 $f(x)=x^2+2x-1$ 相切之二條切線方程式，請將:

   (a) 此二條切線方程式寫出；(5%)
   (b) 二個相切的點寫出；(5%)
   (c) P 與二個相切的點所形成的三角形面積算出。(10%)

3. 如 Fig.1，兩個半圓球（半徑皆為 R）以六顆均勻置放的螺絲由外緣拴住，並由一繩支撐。球內開始裝水至裝滿為止，球頂有一小洞經中空掛鉤連通至大氣。試求出：

   (a) 球內 A、B、C 三點承受的水壓；(6%)
   (b) 每個螺絲於加水後所增加的垂直受力。(5%)
   (c) 若小洞密封，且無空氣殘留，每個螺絲因加水所增加的垂直受力又將為何？(5%)
   (d) 若下半球的底端開一孔，比較球頂小洞密封與不密封時，水流出速率何者較快？(4%)

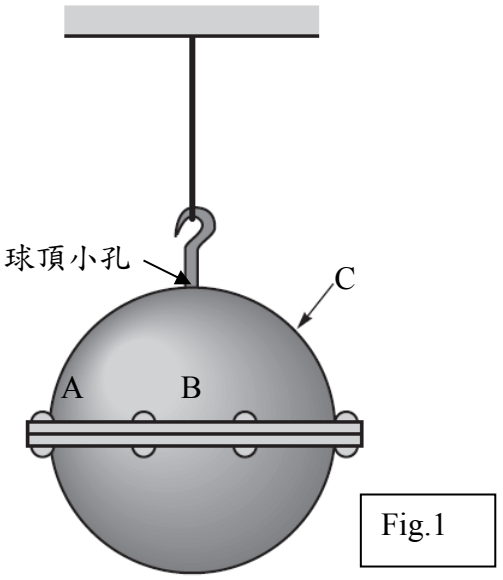

Fig.1

（假設水密度為ρ，重力加速度為 g，大氣壓力為 P_0；A、B 二點均在通過球心之水平面，但 B 在中心而 A 在邊緣；C 點高度在球頂點與此中心水平面的一半處。）

4 一壓縮彈簧受力時會出現兩段不同的力常數，如 Fig.2 所示的施力-變形位移關係

   (i) 當變形從 0 至 S 時，彈簧材料的力常數 k_1=300 牛頓/公尺
   (ii) 當變形從 S 之後，彈簧材料的力常數變為 k_2=450 牛頓/公尺

今有一實驗，以一質量 500 克的物件並以每秒 5 公尺的速度撞擊該彈簧，彈簧必須經過兩段力常數的變形且最終總變形量為 20 公分。假設彈簧第一段及第二段變形過程均是彈性變形且撞擊過程無其他能量的損失，而測試物件最後的速度為零，試問:

   (a) 第一段變形 S 的位移為多少公尺? (10%)
   (b) 測試物件靜止時，彈簧的受力為多少牛頓?(10%)

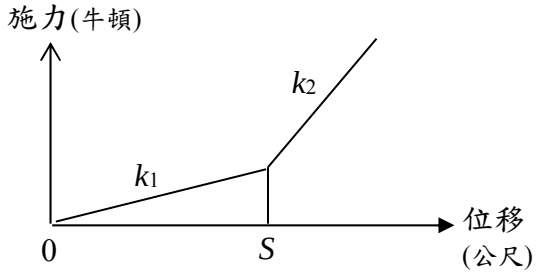

Fig.2

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5　請仔細閱讀下面文章，並據以回答後面的問題。

What Are Tidal Forces (起潮力）?

Centrifugal force (離心力）

As the Earth and moon whirl around their barycenter（共同質心）, centrifugal force arises and is always directed away from the barycenter. All points in or on the surface of the Earth acting as a coherent body acquire this component of centrifugal force. Since the center-of-mass of the Earth is always on the opposite side of the barycenter from the position of the moon, the centrifugal force produced at any point in or on the Earth is always directed away from the moon. The magnitude of this centrifugal force is the same at any point on or beneath the Earth's surface.

It is important to note that the centrifugal force produced by the daily rotation of the Earth on its axis must be completely disregarded in the tidal theory. This element plays no part in the establishment of the differential tide-producing forces.

Gravitational force（萬有引力）

For orbital purposes, Newton's law of gravitation treats two bodies as zero-dimensional points (質點). Thus the gravitational force between the Earth and the Moon is often expressed as:

$$F = G\frac{m_{earth}m_{moon}}{r^2}$$

where m_{earth} and m_{moon} are the masses of the Earth and the Moon, r is the distance between their centers of mass and G is the universal gravitational constant（萬有引力常數）. However, real objects such as planets（行星）and satellites（衛星）have significant non-zero diameters. The force of gravity is significantly different between points on the near sides of two bodies and between points on their far sides.

If the radius the Earth is "w", the point on the Earth that is the closest to the Moon feels a gravitational attraction of:

$$F = G\frac{m_{earth}m_{moon}}{(r-w)^2}$$

while the point farthest from the Moon feels a gravitational attraction of:

$$F = G\frac{m_{earth}m_{moon}}{(r+w)^2}$$

The near point is being pulled more strongly toward the Moon, while the far point is being pulled less strongly, than the center. Relative to the center, which determines the Moon's orbital position, the near point is pulled toward Earth, while the far point is pushed away from Earth. The difference between the centrifugal and these gravitational forces is the tidal force. If one calculates this difference for every point on the Earth, one gets the vectors of the tidal force as shown in Fig.3.

Thus the Earth is stretched along an axis pointing toward the Moon, resulting in a "bulge" on both the near and far sides. The magnitude of this stretching tidal force is approximated by:

$$F = \frac{2Gm_{earth}m_{moon}w}{r^3}$$

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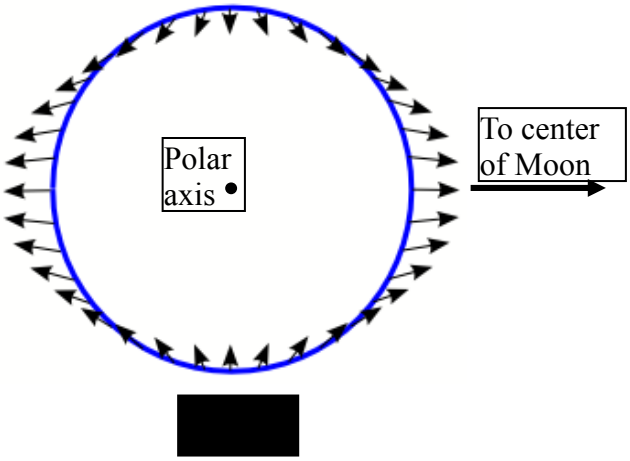

The tidal accelerations（起潮力加速度） at the surface of the Earth generated by the Moon along the Moon-Earth axis is about $1.1 \times 10^{-7} g$, where g is the gravitational acceleration（重力加速度）at the Earth's surface.

Now let's consider that the Earth is also rotating. If the Earth were rotating at the same rate as the Moon's orbital period, the bulge would always point straight at the Moon (Fig.4a). However, the Earth is rotating considerably faster than the Moon's orbital period, and the bulge, which can't move as quickly, is pushed ahead, and points ahead of the Moon's position (Fig.4b). This generates a backward torque (Fig.4c) which gradually slows the rotation of the Earth.

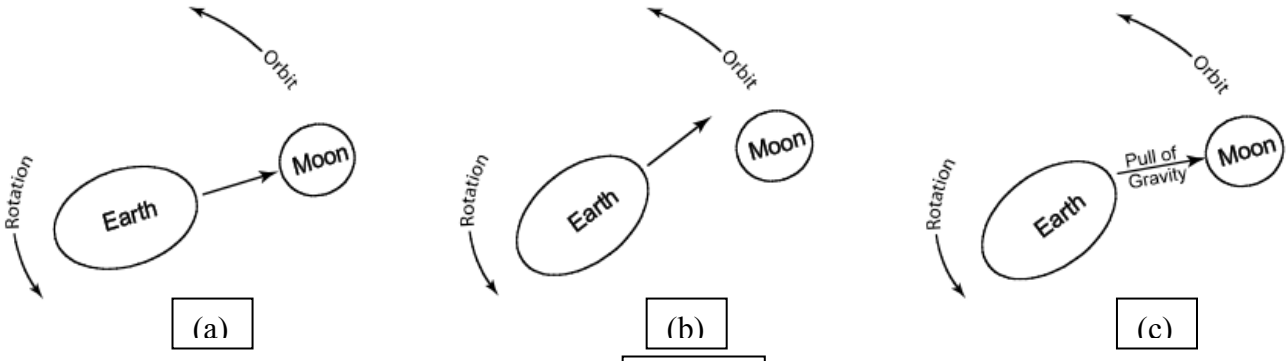

Fig. 4

問題：

(a) 試簡述海水潮汐如何產生。（10%）

(b) 如文中所述，沿地球-月亮軸線方向的起潮力（也就是 Fig.3 中最大的起潮力）才只有重力的 1.1×10^{-7} 倍，試解釋潮水為何仍被掀起？（5%）

(c) 400 萬年前，一年（地球公轉一周）約有四百天，試簡單解釋為何目前約只有 365 天？（5%）