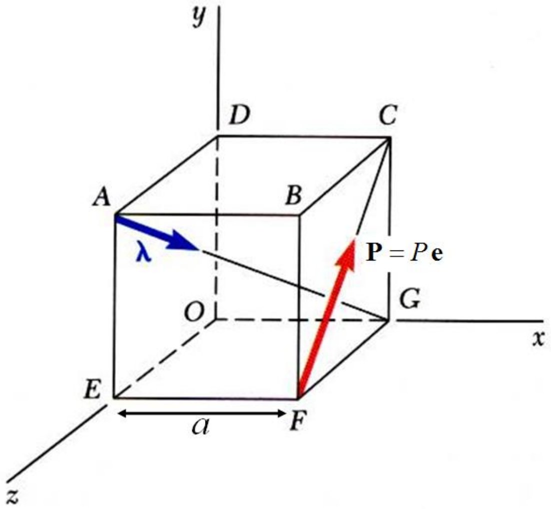
一邊長為 a 的立方體受一力 $\mathbf{P} = P\,\mathbf{e} = P_x\,\mathbf{i} + P_y\,\mathbf{j} + P_z\,\mathbf{k}$ 作用，已知力的大小為 P，如下圖所示，粗體符號表示向量、斜體符號表示純量。其中 $\mathbf{i} = (1, 0, 0)$, $\mathbf{j} = (0, 1, 0)$, $\mathbf{k} = (0, 0, 1)$分別為指向正 x、y 和 z 軸的單位向量。$\boldsymbol{\lambda}$、$\mathbf{e}$ 分別為沿 AG 和 FC 的單位向量，$|\boldsymbol{\lambda}| = |\mathbf{e}| = 1$。

(a) (2%) 試以 P 表示 P_x、P_y 和 P_z。
(b) (3%) 試以 $\mathbf{i}, \mathbf{j}, \mathbf{k}$ 表示$\boldsymbol{\lambda}$。
(c) (5%) 試求 $\mathbf{P}$ 對點 A 的力矩 $\mathbf{M}_A$。
(d) (5%) 若 AG 為固定軸，立方體可繞 AG 旋轉，$\mathbf{P}$ 對 AG 的力矩為 $M_{AG}\boldsymbol{\lambda}$。試求 M_{AG} 。（提示：若一力平行於轉軸，則此力無法使物體對轉軸旋轉，此時力矩為零。）
(e) (5%) 試求 AG 與 FC 的垂直距離。



參考解答

STRATEGY: Use the equations presented in this section to compute the moments asked for. You can find the distance between AG and FC from the expression for the moment M_{AG}.

MODELING and ANALYSIS:

a. Moment about *A*. Choosing x, y, and z axes as shown (Fig. 1), resolve into rectangular components the force $\mathbf{P}$ and the vector $\mathbf{r}_{F/A} = \overrightarrow{AF}$ drawn from A to the point of application F of $\mathbf{P}$.

$$\mathbf{r}_{F/A} = a\mathbf{i} - a\mathbf{j} = a(\mathbf{i} - \mathbf{j})$$
$$\mathbf{P} = (P/\sqrt{2})\mathbf{j} - (P/\sqrt{2})\mathbf{k} = (P/\sqrt{2})(\mathbf{j} - \mathbf{k})$$

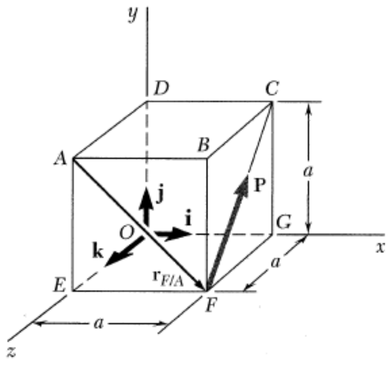


Fig. 1 Position vector $\mathbf{r}_{F/A}$ and force vector $\mathbf{P}$ relative to chosen coordinate system.

You can verify that since AB is parallel to the x axis, M_{AB} is also the x component of the moment $\mathbf{M}_A$.

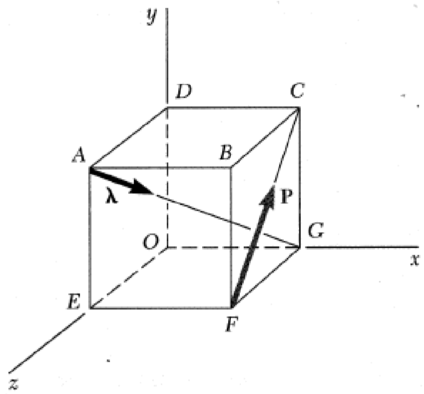


Fig. 2 Unit vector **λ** used to determine moment of **P** about AG.

c. Moment about diagonal *AG*. You obtain the moment of **P** about AG by projecting $\mathbf{M}_A$ on AG. If you denote the unit vector along AG by $\boldsymbol{\lambda}$ (Fig. 2), the calculation looks like this:

$$\boldsymbol{\lambda} = \frac{\overrightarrow{AG}}{AG} = \frac{a\mathbf{i} - a\mathbf{j} - a\mathbf{k}}{a\sqrt{3}} = (1/\sqrt{3})(\mathbf{i} - \mathbf{j} - \mathbf{k})$$

$$M_{AG} = \boldsymbol{\lambda}\cdot\mathbf{M}_A = (1/\sqrt{3})(\mathbf{i} - \mathbf{j} - \mathbf{k})\cdot(aP/\sqrt{2})(\mathbf{i} + \mathbf{j} + \mathbf{k})$$

$$M_{AG} = (aP/\sqrt{6})(1 - 1 - 1) \quad M_{AG} = -aP/\sqrt{6} \blacktriangleleft$$

Alternative Method. You can also calculate the moment of **P** about AG from the determinant form:

$$M_{AG} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x_{F/A} & y_{F/A} & z_{F/A} \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} 1/\sqrt{3} & -1/\sqrt{3} & -1/\sqrt{3} \\ a & -a & 0 \\ 0 & P/\sqrt{2} & -P/\sqrt{2} \end{vmatrix} = -aP/\sqrt{6}$$

d. Perpendicular Distance between *AG* and *FC*. First note that **P** is perpendicular to the diagonal AG. You can check this by forming the scalar product $\mathbf{P}\cdot\boldsymbol{\lambda}$ and verifying that it is zero:

$$\mathbf{P}\cdot\boldsymbol{\lambda} = (P/\sqrt{2})(\mathbf{j} - \mathbf{k})\cdot(1/\sqrt{3})(\mathbf{i} - \mathbf{j} - \mathbf{k}) = (P\sqrt{6})(0 - 1 + 1) = 0$$

You can then express the moment M_{AG} as $-Pd$, where d is the perpendicular distance from AG to FC (Fig. 3). (The negative sign is needed because the rotation imparted to the cube by **P** appears as clockwise to an observer at G.) Using the value found for M_{AG} in part c,

$$M_{AG} = -Pd = -aP/\sqrt{6} \qquad d = a/\sqrt{6} \blacktriangleleft$$

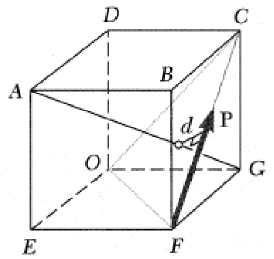


Fig. 3 Perpendicular distance d from AG to FC.

REFLECT and THINK: In a problem like this, it is important to visualize the forces and moments in three dimensions so you can choose the appropriate equations for finding them and also recognize the geometric relationships between them.