一矩形板由 A 與 B 兩處的拖架以及一線材 CD 支撐。已知線材施加一張力於點 C，此張力可表示為向量 $\mathbf{P} = P\,\mathbf{e} = P_x\,\mathbf{i} + P_y\,\mathbf{j} + P_z\,\mathbf{k}$，其大小 P 為 200 N。粗體符號表示向量、斜體符號表示純量。其中 $\mathbf{i} = (1, 0, 0)$, $\mathbf{j} = (0, 1, 0)$, $\mathbf{k} = (0, 0, 1)$分別為指向正 x、y 和 z 軸的單位向量。$\mathbf{e}$ 為沿 CD 的單位向量，$|\mathbf{e}| = 1$。

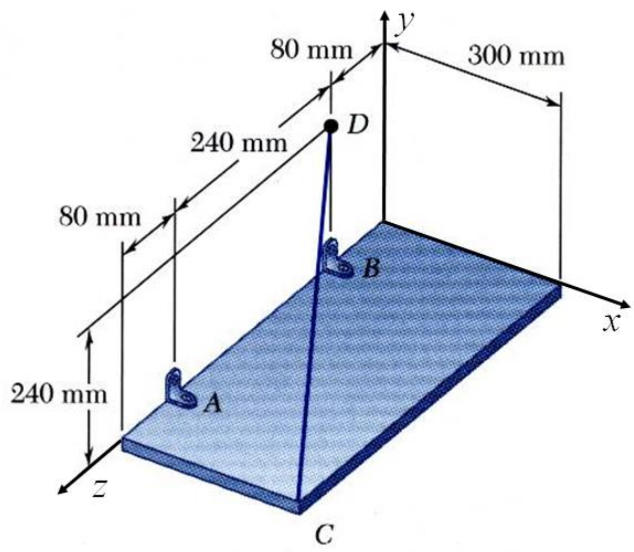
(a) (3%) 試以 $\mathbf{i}, \mathbf{j}, \mathbf{k}$ 表示 $\mathbf{e}$。

(b) (2%) 試求 P_x、P_y 和 P_z。

(c) (5%) 試求 $\mathbf{P}$ 對點 A 的力矩 $\mathbf{M}_A$。

(d) (5%) 若 AB 為固定軸，矩形板可繞 AB 旋轉，$\mathbf{P}$ 對 AB 的力矩為 $M_{AB}\,\mathbf{k}$。試求 M_{AB} 。（提示：若一力平行於轉軸，則此力對轉軸的力矩為零。）

(e) (5%) 試求 AB 與 CD 的垂直距離。



参考解答

STRATEGY: The solution requires resolving the tension in the wire and the position vector from A to C into rectangular components. You will need a unit vector approach to determine the force components.

MODELING and ANALYSIS: Obtain the moment $\mathbf{M}_A$ about A of the force $\mathbf{F}$ exerted by the wire on point C by forming the vector product

$$\mathbf{M}_A = \mathbf{r}_{C/A} \times \mathbf{F} \qquad (1)$$

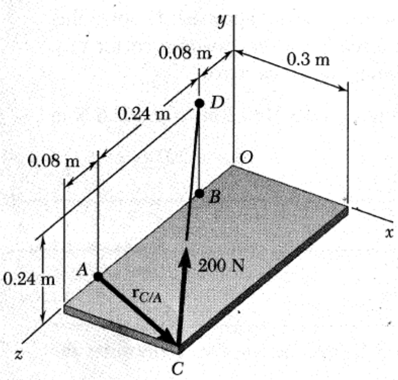


Fig. 1 The moment $\mathbf{M}_A$ is determined from position vector $\mathbf{r}_{C/A}$ and force vector $\mathbf{F}$.

where $\mathbf{r}_{C/A}$ is the vector from A to C

$$\mathbf{r}_{C/A} = \overrightarrow{AC} = (0.3\text{ m})\mathbf{i} + (0.08\text{ m})\mathbf{k} \qquad (2)$$

and $\mathbf{F}$ is the 200-N force directed along CD (Fig. 1). Introducing the unit vector

$$\boldsymbol{\lambda} = \overrightarrow{CD}/CD,$$

you can express $\mathbf{F}$ as

$$\mathbf{F} = F\boldsymbol{\lambda} = (200\text{N})\frac{\overrightarrow{CD}}{CD} \qquad (3)$$

Resolving the vector $\overrightarrow{CD}$ into rectangular components, you have

$$\overrightarrow{CD} = -(0.3\text{ m})\mathbf{i} + (0.24\text{ m})\mathbf{j} - (0.32\text{ m})\mathbf{k} \qquad CD = 0.50\text{ m}$$

Substituting into (3) gives you

$$\mathbf{F} = \frac{200\text{N}}{0.50\text{m}}[-(0.3\text{ m})\mathbf{i} + (0.24\text{ m})\mathbf{j} - (0.32\text{ m})\mathbf{k}]$$
$$= -(120\text{ N})\mathbf{i} + (96\text{ N})\mathbf{j} - (128\text{ N})\mathbf{k} \qquad (4)$$

Substituting for $\mathbf{r}_{C/A}$ and $\mathbf{F}$ from (2) and (4) into (1) and recalling the relations in Eq. (3.7) of Sec. 3.1D, you obtain (Fig. 2)

$$\mathbf{M}_A = \mathbf{r}_{C/A} \times \mathbf{F} = (0.3\mathbf{i} + 0.08\mathbf{k}) \times (-120\mathbf{i} + 96\mathbf{j} - 128\mathbf{k})$$
$$= (0.3)(96)\mathbf{k} + (0.3)(-128)(-\mathbf{j}) + (0.08)(-120)\mathbf{j} + (0.08)(96)(-\mathbf{i})$$

$$\mathbf{M}_A = -(7.68\text{ N·m})\mathbf{i} + (28.8\text{ N·m})\mathbf{j} + (28.8\text{ N·m})\mathbf{k} \blacktriangleleft$$

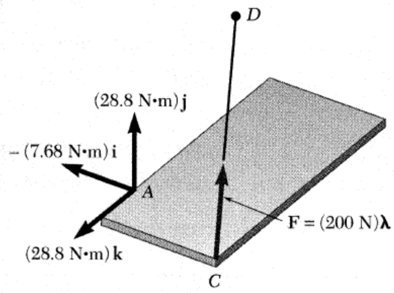


Fig. 2 Components of moment $\mathbf{M}_A$ applied at A.

Alternative Solution. As indicated in Sec. 3.1F, you can also express the moment $\mathbf{M}_A$ in the form of a determinant:

$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_C - x_A & y_C - y_A & z_C - z_A \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.3 & 0 & 0.08 \\ -120 & 96 & -128 \end{vmatrix}$$

$$\mathbf{M}_A = -(7.68\text{N·m})\mathbf{i} + (28.8\text{ N·m})\mathbf{j} + (28.8\text{ N·m})\mathbf{k} \blacktriangleleft$$

REFLECT and THINK: Two-dimensional problems often are solved easily using a scalar approach, but the versatility of a vector analysis is quite apparent in a three-dimensional problem such as this.